# The role of thermodynamic efficiency in Portugal's economic growth between 1960 and 2014: Development

and application of a macroeconomic model with useful exergy

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### Abstract

Energy is underestimated by the economic growth models used to outline economic and sustainability policies, which are unable to: 1) estimate the economic impact of energy crises; 2) explain growth only with traditional factors of production - capital and labor. As has been shown, the contribution of energy to the economy must take into account the useful stage of energy uses, as well as the capacity of the latter to perform physical work (exergy). Thermodynamically consistent models, such as the *MAcroeconometric Resource COnsumption* for the United Kingdom (MARCO-UK), demonstrate the strong link between useful exergy and economic growth.

The objective of this work is the adaptation and application of the MARCO-UK model to Portugal 1960-2014 (MARCO-PT). This process includes: a) collecting economic and energy data; b) econometrically test (cointegration) the validity of the relationships between MARCO-UK variables for the Portuguese reality; c) adapt and / or reformulate these relations in order to obtain a good model of the Portuguese economy. After some adjustments, the model is used for counterfactual simulations, isolating the effects of useful exergy (among other variables) on Portugal's economic growth.

The results obtained with MARCO-PT reinforce the conclusions of MARCO-UK, namely that the increasing availability of useful exergy - resulting from gains in final-to-useful exergy efficiency - has been a major driver of economic growth in Portugal (40% of GDP growth in 2014), exceeding capital investment and human labor. By including energy in a thermodynamically consistent way, the MARCO-PT model provides new perspectives for sustainable economic development.

**Keywords**: Useful exergy; Cointegration; Macroeconomic model; Economic growth; Energy efficiency; Sustainable development.

### Introduction

Since the industrial revolution, nations have had economic development as their major global objective. This serves to measure the progress of nations which, together with good public policies, measures social development as well. Thus, Western society, and much of the world, uses the growth of gross domestic product (GDP) as a gauge of the country's "quality". It is not a perfect measure, of course, but over the past century it has played a major role in the advancement of societies. Such development, as I intend to show, was strongly related to the different technological revolutions, in particular to the way in which energy was used throughout the economy.

It is still very common for energy not to be considered a fundamental part of a country's production. Thus, it is not included in the vast majority of macroeconomic models that govern, to a large extent, national policies. Some economists even claim that economic growth is largely independent of energy use [Warr et al., 2006]. Therefore, it is possible to affirm that traditional models are not capable of describing economic development. This is due to two fundamental reasons. First, the fact that they do not consider energy as a primary productive factor, which means that they are not able to explain the impact of energy crises on the economy. Second, considering only capital and human labor, a large portion of the growth remains unexplained, and therefore, a residue that is not explained by the model (Total Productivity of Factors -TPF) remains.

In contrast to what the neoclassical theory states, there is a current in the literature called ecological economics, a theory was developed in which energy plays a major role in the growth of a nation. This theory deals with economics as a subsystem of a larger system, in which interactions between economics and energy are based on the laws of physics. Thus, energy contributions are essential for economic production as real processes always require energy and cannot be described without it [Santos et al., 2018]. These authors state that the economic system is inserted in the environment and that it exchanges matter and energy with it. Bearing this in mind, this theory argues that economic thinking should be based on the laws of physics, namely the laws of thermodynamics, as these are the laws that "govern" the referred exchanges of matter and energy. There is a lot of work developed that proposes that energy should be taken into account when developing macroeconomic models. In fact, it is, above all, seen as a key piece of explanation for the economic growth of nations.

In today's societies, energy sources (primary energy) are usually not used directly for purposes useful to consumers. Conversions are needed along the energy chain. Very few people nowadays burn fossil fuels to heat their homes. These fuels - for example, coal - are used to produce electricity (final energy) in thermoelectric plants and it will be this electricity that people will use in their homes - for example, a radiator that converts electrical energy into heat (useful energy). This evolution is due to the fact that man is able to obtain energy today in more concentrated and more easily transportable forms.

Of course, energy efficiency and demand in a country will be important, as it will be these data that will regulate the amount of primary energy needed. Usually, the energetic approach for this evaluation is based on the 1st Law of Thermodynamics. However, this way of evaluating is not completely reliable because, in addition to treating different types of energy as equal (not being so), it does not illustrate that not all energy inputs are capable of producing work. To better understand this difference, it is necessary to clarify what is meant by exergy, useful exergy, and efficiency.

Having two systems, one reference, called the environment and the other, considered the system of interest, exergy will be the maximum theoretical work that can be obtained through the interaction of the two systems to achieve equilibrium[Grubler et al., 2012].

The following example serves to illustrate this concept by differentiating exergy from the concept of energy itself. Imagine a small, isolated room that contains a container of gasoline. Fuel is lit and it burns until all the liquid is consumed. The result of this experiment will be a slight increase in the room's air temperature (now containing air and the products of combustion). Assuming that the room is in fact well insulated, the total amount of energy has been maintained. What changed was the "quality" of the energy inside the room. Gasoline before burning has a greater potential to perform useful tasks, compared to the mixture of air and slightly heated combustion products. For example, gasoline could have been used to produce electricity, or to drive a car. The mixture of heated gases, on the other hand, would not have the capacity to accomplish almost anything, except to improve the thermal sensation inside the room. Indeed, the initial potential of gasoline (its exergy) was destroyed during combustion. Despite the energy being maintained, exergy is always destroyed in all energy conversion processes (2nd Law of Thermodynamics) [Brockway et al., 2016].

To better master this concept, it is still necessary to distinguish between first law efficiency (related to energy) and second law efficiency (exergetic efficiency). In order to do this, let us use an example again. Consider a water heater with an energy efficiency (1st law) of 80% i.e. for each unit of energy input (e.g. Joule), the device supplies 0.8 units of energy. This high efficiency could suggest that there is little room for improvement, since the efficiency of the device is very close to 100%. However, that conclusion would be wrong. The value of 80% only refers to the specific process of the operation in question, in this case, converting fuel into heat. Bearing in mind that the temperatures involved in the combustion of fossil fuel are much higher than those required by the consumer at home, it appears that this process does not suit the needs in question, i.e. it is an inefficient use of fuel and the device itself.

Thus, the present study used useful exergy to try to explain economic development, due to the fact that this metric captures the contribution of energy to the economy and the well-being of society in the best way. The exergy destroyed along the energy chain has no economic value, but the useful exergy at the end of the chain is what produces all the goods and services in an economy. Therefore, improving second-law conversion efficiencies at all stages of the chain will allow more useful exergy to reach the consumer, resulting from the same amount of primary energy. This increase in available useful energy is what will bring economic development and not just an increase in the energy output of the conversion devices. In short, it will be the productive part of energy flows (useful exergy) that should be the focus of macroeconomic models [Serrenho et al., 2016].

As mentioned earlier, traditional models of economic growth analysis do not consider energy as a relevant variable in this analysis. They limit themselves to considering it as another indirect product of the production process. More recent models - associated with the theory of ecological economics - assume that energy plays a major role in the growth of a country's product. However, not all energy plays a productive and useful role in the economy. Thus, recently there have been authors proposing macroeconomic models, considering energy as a factor of production - together with capital and human labor - in which the metric used to account for energy is useful exergy. Of the works carried out in this area, the most complete and developed is the MARCO-UK model [Sakai et al., 2019]. This model had as main results and conclusions, evidences that the efficiency of conversion of final exergy into useful is an essential engine of the economic growth, having been responsible for about 25% the growth of the GDP of the United Kingdom of the last 50 years. In addition, they said that this efficiency had two distinct sources: 1) technological progress and 2) increased demand for energy services. Finally, the MARCO-UK model came to the conclusion that, on the one hand, human labor is a productive factor with little relevance to economic growth and, on the other, that energy and capital complement each other in the great influence they have on UK economic development. However, this model - like any other - has limitations. The present work aims, based on MARCO-UK, to overcome some of these limitations. In particular, it seeks to apply the model to another country (Portugal), in order to confirm or reject the conclusions drawn from the application of the model to the United Kingdom. In addition, this work aims to develop the statistical methods used in the British model.

### Methods

The first step in the analysis was the identification of each of the 75 variables included in the construction and implementation of the MARCO-UK model [Sakai et al., 2019], obtained from international databases such as the UK Office for National Statistics, the World Bank, Penn World Tables, and the United Nations. These variables are divided into macroeconomic variables, at the country level (GDP, capital stock, exports / imports, etc.), macroeconomic variables at the sectoral level (expenditure in industry, residential sector, etc.), prices (price index consumer, inflation, etc.), monetary variables (exchange rate, interest rates, etc.), energy variables (consumption, prices, efficiency, exergy etc.), and polluting emissions (CO2 per capita, territorial and from the perspective consumption). An exhaustive list of all variables included in the MARCO-UK model can be found in [Sakai et al., 2019 - Supplementary Information].

Once the necessary variables for the implementation of the MARCO-UK model for the Portuguese reality were identified, a wide research was carried out to collect the data necessary to achieve this implementation, through a process of consulting international and specific databases of the Portuguese economy, as well as individual published articles, in order to obtain time series equivalent to those used in the MARCO-UK model. This collection was made with the objective of obtaining a set of time series for the Portuguese economy that not only reflected as much as possible the set of data used in the MARCO-UK model, but also covered a period as long as possible, given the availability of the data , for this country.

The MARCO-UK model [Sakai et al., 2019] consists of 57 equations, of which 30 correspond to identities, or definition relationships between variables. These identities result from definitions inserted in the system of national accounts, and for that reason they are valid not only over time, but also for all countries. As such, these identities are applicable to both the UK and Portugal. Two examples of these identity relations implemented in the MARCO-UK model are the definition of Gross Domestic Product (GDP) from the expenditure side, or from the income side, represented below:

 $Y = C + I + G + X - M \quad (1)$  $Y = W + YG + YF \quad (2)$ 

Where Y corresponds to GDP, which in terms of expenditure is defined as the sum of consumption expenditure by households (C) and by the government (G), investment expenditure (I), and the balance between exports (X) and imports (M). In terms of income, the same GDP is defined as the sum of workers' income, or wages (W), government income, or taxes less subsidies (YG), and the income of companies and others (YF). Both of these relationships are valid and produce the same result as GDP Y for any moment in the economy.

The remaining 27 equations that make up the MARCO-UK model differ from the identities described above. These are based on empirical observations specific to the UK economy, and capture the structure of that particular economy. These "behavioral" or "stochastic" equations contain parameters estimated by rigorous econometric methods (e.g. least squares methods, cointegration), and relate key variables of the model, such as capital, energy prices, human labor, among others. For example, human labor (L), in

the MARCO-UK model, is a function of GDP (Y), service capital (K\_SERV) and useful exergy (UEX\_TOT) - equation 3.

 $L_t = f(Y_t, K\_SERV_t, UEX\_TOT_t)$ (3)

In the process of adapting and implementing the MARCO-UK model to the Portuguese reality, the most important step will be to test the validity (or not) of the stochastic equations inserted in the MARCO-UK model to the equivalent data collected for the Portuguese economy. In this exercise, it is expected that, for each stochastic equation, one of three results will be obtained:

- Case 1: the stochastic equation used in the MARCO-UK model is perfectly suited to the data obtained for Portugal, without the need for changes or corrections to reflect idiosyncrasies of the Portuguese economy;
- Case 2: the stochastic equation used in the MARCO-UK model captures, in general, the relationship between the equivalent variables for Portugal, but requires changes or specific corrections in order to more accurately reflect the Portuguese economy;
- Case 3: the stochastic equation used in the MARCO-UK model is not at all suited to Portuguese reality;

Each of the cases listed above has consequences for the way in which the adaptation and implementation of the MARCO-UK model for Portugal is conducted. In Case 1, the MARCO-UK model stochastic equations are used in the implementation of the model for Portugal, using only its form, ie, using all the terms of the MARCO-UK model equation, except for their coefficients, which have been re-estimated. In Case 2, the general formula of the MARCO-UK model stochastic equation is used, with only possible changes/corrections that may involve the reestimation of some coefficients, or the inclusion of specific coefficients for a given year(s), in order to capture specific behaviors of the Portuguese economy. Finally, in Case 3, a stochastic equation is estimated, econometrically, from scratch, trying to relate the same variables, for Portugal, present in the equivalent econometric equation inserted in the MARCO-UK model.

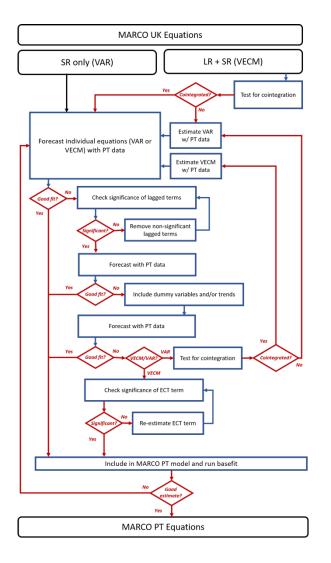


Figure 1 – Flow chart representing the estimation of individual stochastic (econometric) equations to be included in the MARCO PT modelling framework, based on the stochastic (or econometric) equations that make up the MARCO UK modelling framework.

Figure 1 illustrates the development process of the econometric equations that will formulate the MARCO-UK model, always starting from the equations already present in the model for the United Kingdom. The stochastic equations of the MARCO-UK model are tested by replacing the time series corresponding to the British economy with data obtained for Portugal. Initially, the Portuguese time series are projected using the functionality of each of the equations developed for the MARCO-UK model (the form of the equation, without the original coefficients).

If a stochastic equation for the United Kingdom is able to estimate the corresponding dependent variable for the Portuguese economy (between 1960-2014), then that same equation is included directly in the new MARCO-PT model. This case corresponds to the first case mentioned above.

If, however, an equation of the MARCO-UK model when estimating the corresponding dependent variable for the Portuguese economy, shows significant differences from the real values of that dependent variable, then the equation will enter an iterative correction process.

Finally, if after the correction process, the equation continues to not satisfactorily estimate the dependent variable, the third case is verified. The equation needs to be modified more profoundly, namely the long-term dynamics term.

After all econometric equations go through the iterative method and are able to correctly estimate the variables of the Portuguese economy, they are introduced in the new MARCO-PT model, together with the identity equations.

Then, the model is solved - in a dynamic way following the iterative method of Gauss- Seidel [Varga, 2009]. In а stochastic simulation, the model's equations are solved so that their residuals correspond to random errors and, optionally, that the exogenous coefficients and variables of the model also vary randomly. In these simulations, the resolution of the model generates a distribution of results for the endogenous variables in each period. The distribution is approximate by solving the model repeatedly, often, using different samples for the random components of the model, and then calculating the statistics taking into account all the results obtained.

If the *basefit* model is not correctly estimating data from the Portuguese economy, it will be necessary to re-evaluate each of the econometric equations that make up the model according to the iterative process mentioned above. All of this is repeated until a suitable *basefit* model is obtained. When the MARCO-PT model is validated, having a basefit model that adequately estimates the data of the Portuguese economy, it will perform counterfactual simulations - just as in the MARCO-UK model [Sakai et al., 2019]. These simulations have as main objective to demonstrate, in an elucidative way, what is the role of exergetic efficiency (final to useful) in the economic growth of Portugal. The result of these simulations will serve to understand how much the Portuguese economy could grow compared to the growth shown in the basefit model. The six simulations will set the following variables:

# Thermodynamic efficiency from final to useful exergy (EXEFF\_FU);

- 2- Total final energy consumption (sum of consumption by families FEN\_C, industry FEN\_IND, and other FEN\_OTH);
- 3- Investment (I);
- 4- Human labor (in number of workers L);
- 5- Energy prices (paid by families P\_EN\_C, industry P\_EN\_IND, and others P\_EN\_OTH);
- 6- Total useful exergy (UEX\_TOT);

# **Results and discussion**

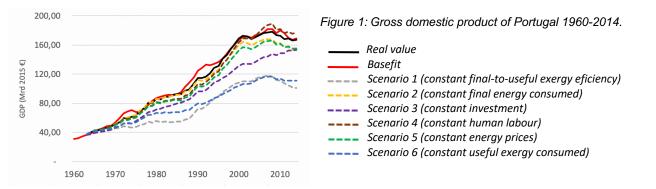


Figure 1 shows the graph referring to Portugal's GDP (Y) estimate - between 1960 and 2014 - for each of the counterfactual simulations. In the graph it is possible to identify the Y projection of the basefit solution by comparing it with the Y evolution of the counterfactual simulations. The chart starts its values in 1963 and not in 1960 due to the lagged terms.

It is clear that, forcing the EXEFF\_FU variable to maintain its 1963 value (approximately 14.5%) throughout the study period, it leads to an abrupt drop in gross domestic product, when compared to real GDP. In this simulation, the GDP of the basefit solution in 2014 represents more than half (approximately 60%) of the GDP that would be obtained with the efficiency kept constant.

In relation to the setting of the final useful exergy at values from 1963 (approximately 27 PJ) until 2014, this leads to the fact that the GDP estimate for

Portugal is about 67% lower than the basefit estimate in 2014.

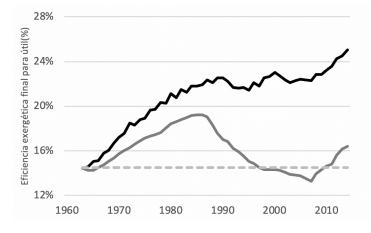
Analyzing these two facts together, we understand the vitality of energy consumption - measured at the useful stage and in exergetic terms - for Portugal's economic growth.

In order to better understand and identify the contributions that each of the variables, present in counterfactual simulations, have in Portugal's economic growth, the table (table 1), present in [Sakai et al., 2019], in which they are the differences between the annual averages of the GDP growth rate - estimated by the basefit solution of the MARCO-PT model - are shown and the GDP growth rate - estimated by each of the counterfactual simulations.

Scenario	Constant	Annual contribution to growth of GDP					
Scenario	variable	1963-1973	1973-1985	1985-1994	1994-2005	2005-2014	1963-2014
Basefit GDP growth rate		4,76%	3,75%	3,76%	2,81%	-0,42%	3,01%
Scenario 1	Exergy efficiency	2,39%	2,48%	-1,45%	0,13%	1,08%	1,01%
Scenario 2	Final energy	-0,12%	0,13%	0,38%	0,10%	0,38%	0,16%
Scenario 3	Investment	0,96%	0,23%	0,23%	0,42%	-1,42%	0,12%
Scenario 4	Labour supply	0,25%	0,43%	-0,10%	-0,80%	-0,41%	-0,11%
Scenario 5	Energy prices	0,55%	0,38%	0,15%	-0,26%	-0,04%	0,16%
Scenario 6	Useful exergy	1,41%	1,46%	0,97%	0,37%	-0,19%	0,84%

Table 1: Contributions to the GDP growth of selected variables. Light gray: periods when the contribution to GDP growth was between 0.25-1.00%/year. Medium gray:periods when the contribution to GDP growth was between 1.00-2.00% / year. Dark gray: periods when the contribution to GDP growth was> 2.00% / year.

Taking all these results into account - which are in line with the results obtained by applying the MARCO-UK model to the United Kingdom [Sakai et al., 2019] useful exergy has a greater impact on economic growth than the final energy consumed or than energy prices. Energy consumption, measured in its useful phase and in exergetic quantities, presents itself as the energy variable that is most related and affects economic growth. This fact corroborates the arguments of the authors - pioneers in the area of exergetic economics - Robert Ayres and Benjamin Warr [Ayres & Warr, 2005], who defend the importance of useful exergy in the economy, claiming that it is useful exergy (or useful work) the energy metric that best captures the contributions of energy flows to the economy. In the specific case of Portugal,



the results of the present thesis add evidence to previous works. Namely, [Serrenho et al., 2016] who found that the intensity of useful exergy (useful exergy / GDP) is considerably stable over long periods of time - while the intensity of the final energy decreased over the same period of time - and [Santos et al., 2018] who found that aggregate production functions statistically significant and economically plausible - are obtained from the test of cointegration only relationships between economic product and possible factors of production when a measure of useful exergy is included in the cointegration (that is, in the set of variables for which a relationship is being tested).

It is possible to divide the contribution of the aggregate efficiency of converting final exergy into useful into two 1) contributions resulting components: from technological advances introduced in the economy and which led to an increase in exergetic efficiency (technological progress); 2) contributions due to the increased demand for useful exergy in the economy (demand for energy services). Both can be identified by looking at the estimates for the final exergetic efficiency to be useful in scenario 1 (constant exergy efficiency) and in scenario 6 (constant useful exergy) and comparing them with the basefit model estimates – figure 2.

Figure 2: Gains in final energy efficiency for useful due to two components: technological progress and increased demand for energy services. Basefit (black line); scenario 6 (constant total useful exergy) (gray line); scenario 1 (final exergetic efficiency for constant use) (dashed line).

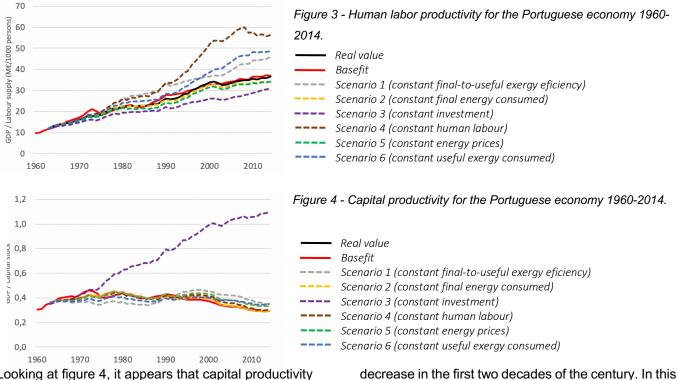
Thus, it is clear, when analyzing figure 2 that technological progress - which leads to an increase in final exergetic efficiency to useful - played a major role in Portugal's economic growth. However, this only happened in the first half of the study period (also shown in table 1). As mentioned earlier, this period in history coincides with the fastest growth in energy efficiency in Portugal, due to the country's massive electrification. In the second half of the study period, the sharp growth in efficiency stopped, with a slightly accelerated increase mainly driven by the increase in demand for energy services. Despite this increase in demand, there was a certain stagnation in terms of efficiency due to, for example, the increased use of the car (energy service associated with low exergetic efficiency).

Figures 3 and 4 show, respectively, the scenarios around the productivity of human labor and capital. In Portugal, the productivity of human labor has grown steadily over the past 50 years, as can be corroborated through the projections of the basefit model (black line). Evidently, in scenario 4 - where human labor is constantly maintained - its productivity

grows significantly. However, keeping the final energy consumption or energy prices constant there is no major change in the growth of the productivity of human labor, while keeping the investment constant leads to a fall in labor productivity. These results are in line with the findings made in [Sakai et al., 2019], when applying the MARCO-UK model to the United Kingdom. In addition, for Portugal it is also understood that there is a somewhat weak relationship between human labor and gross domestic product. Human labor has a much smaller impact on economic growth than capital (i.e., investment) – figure 3 and table 1.

On the other hand, scenarios 1 and 6, are a novelty when comparing the results of the relationship between human labor, consumption of useful exergy and exergy efficiency in the economy of Portugal and that of the United Kingdom. Taking into account the simulations carried out by the MARCO-UK model, human labor goes down considerably in scenarios where both useful exergy and exergy efficiency are kept constant. For Portugal, these same simulations resulted in an increase in the productivity of human labor.

case - of capital productivity - the results of this study



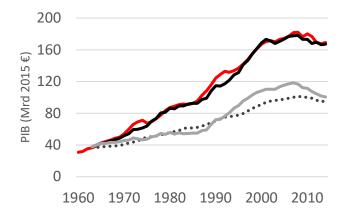
Looking at figure 4, it appears that capital productivity has grown steadily over the past 50 years, with only a

are comparable to the results obtained by the work developed for the United Kingdom [Sakai et al., 2019].

As presented above, neoclassical theory defends models of economic growth that recognize only contributions from two factors of production - capital and human labor. It has generally been published that the role of these two factors is insufficient to explain the economic growth that, in fact, occurs in developed countries. This is, therefore, also the case in Portugal, where counting the contributions of the two mentioned factors of production - both with the weight corresponding to its share of receipts - is about 40% of the GDP growth unexplained.

 $Y = A \cdot K^{0.3} \cdot L^{0.7} \tag{4}$ 

In this way, the PTF's contribution to economic growth during the study period (1963-2014) can be measured,



In figure 5 it is presented how the exogenous technological progress, attributed to PTF in the neoclassical model Cobb-Douglas, is endogenized in the MARCO-PT model due to the inclusion of the variable of conversion efficiency from final exergy to useful. The impact it has on Portugal's economic growth, in light of a neoclassical aggregate production function, by keeping PTF constant during the period of 1963-2014, is well captured by the MARCO-PT model,

# Conclusion

The development of this model (MARCO-PT) demonstrated the central role of energy in Portugal's economic growth. As for the UK economy [Sakai et al., 2019], it was also clear for the Portuguese economy

Previous work has suggested that the final exergetic efficiency for useful can be considered as a relevant approximation of the total factor productivity (TFP), for the case of the economy of Portugal [Santos et al., 2020]. Applying the MARCO model to Portugal, these considerations are reinforced. Modeling the economic output of Portugal, using an aggregate production function common to neoclassical theory - as is the case of the Cobb-Douglas function (equation 4) - and using the elasticities of the constant factors equal to the average of their income in relation to the GDP (approximately 0.3 for capital and 0.7 for human labor), residual TFP A can be calculated.

calculating the same production function (equation 4), but this time keeping factor A constant during the study period. Figure 5 shows the results graphically

Figure 5 – Economic output (real GDP) for the Portuguese economy 1960-2014.

Basefit		
Real GDP		

— Scenario 1 (constant final-to-useful exergy eficiency)

Equation 1 (Cobb-Douglas function)

when in this the final energy efficiency variable for useful it is kept constant. This fact corroborates the statements of [Ayres & Warr, 2005; Ayres & Warr 2006] and the statistical evidence from [Santos et al., 2020], that the technological change assumed in the neoclassical theory - represented by PTF - can be explained after all by the energy conversion efficiency, especially in the final-to-useful stage, measured in exergy terms.

that energy plays a much more important role in economic growth than that suggested by its cost share (5-10%). Thus, investment in policies that promote increased exergetic efficiency (and not just the 1st law energy efficiency) should start to be a reality, taking into account that there is evidence that this investment would result in greater economic growth. the results of this thesis - applied to Portugal - lead us to believe that the presence of exergetic elements in macroeconomic models is essential, given the evidence found, which further strengthens the MARCO model, as it was applied to two different economies, presenting similar results in terms of the direct influence of exergetic efficiency and the use of useful exergy in economic growth.

Developing macroeconomic models that address energy efficiency and energy services will enable a better understanding of the role of energy in the economy and will also provide relevant data to policy makers. After developing this thesis, there is a possibility to continue improving the MARCO model. To this end, it would be possible to take 3 concrete steps: 1) Apply the MARCO model to more economies or groups of economies. 2) In the methodology, use the KPSS test (Kwiatkowski – Phillips – Schmidt – Shin) which, - in addition to the tests used in this work - would facilitate the distinction between stationary, non-stationary series and series that are not distinguishable with just one test. 3) Adopt more than one cointegration relationship between variables, whenever the tests so indicate, instead of just one. 4) test the variables included in a long-term relationship of the model in terms of its Granger causality.

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